A Superior Alternative -- Using the Computer to Determine Yield on an Apartment Investment-- A Case Study-- 1972

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Date of Release: April 10, 1972
Submitted to: Appraisal Journal
Seminar at Urbana-Champaign: April 20, 1972

Acknowledgment: We gratefully acknowledge the invaluable assistance of James Zytko in the development of our IRR-PV real estate investment analysis computer program.
A SUPERIOR ALTERNATIVE--USING THE COMPUTER TO DETERMINE
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The purpose of this article is two-fold. First, we will seek to demonstrate to the reluctant the ease and efficacy of using the computer to calculate a more accurate yield (rate of return on equity) than can be produced using the Ellwood or Inwood tables. Second, by taking the reader through an actual case we hope to reveal information about college-student apartments which will prove of value to those who are interested in that segment of the housing market.

Valuation experts are in agreement that the selection of the capitalization rate represents the key element in the capitalization of income process, and that rate of return to equity demanded by the prudent investor is the most appropriate capitalization rate for apartment investors. It is generally accepted that such capitalization rates are, of course, based upon actual or expected yields and the concepts should be equivalent. Everyone should be familiar with the monumental work done by L. W. Ellwood with the publication of Ellwood Tables for Real Estate Appraising and Financing which represents an important modification of conventional band-of-investment methods for determining capitalization rates.
Many valuable articles have been written which expand upon and clarify the use of the Ellwood Tables and their underlying concepts. For our purposes it suffices to say that Ellwood's Tables are essentially an adaptation of the Inwood compound interest tables to select factors to modify the basic 'R' capitalization rate by weighting it for the effect of the amortization term; contract interest rate; holding period; minimum acceptable yield rate; equity build-up (discounted at the equity-yield rate); and the effect of appreciation or depreciation of value to date of sale.

It must be understood that valuations and yields calculated by use of Ellwood rates, Inwood coefficients, or the corporate financial analyst's internal-rate-return method will be identical, given the same input information (or assumptions). Therefore it is of decisive importance that the user of computer analysis understand the underlying concepts taught by Ellwood in the use of the Inwood compound interest tables. Otherwise, he is a mere robot-extension of the computer program and his input data. Though the Ellwood method has been criticized for treating cash flows as constant and level throughout the holding period; overemphasizing an 8-10 year holding period; and ignoring the impact of IRS taxes on the annual cash flows and the reversion; such criticism are irrelevant to the validity of the conceptual framework.
the new revision of the Elwood Tables have overcome the shortcoming of an unchanging cash flow by introducing a 'J', income adjustment factor, to find changes in both income and property value during the selected projection term. It is hoped we have made it clear that though we advocate the use of the computer, we firmly support Ellwood in his statement,

"The appraiser cannot make an intelligent selection of technique unless he knows exactly what the assumptions are and how they affect his result. Moreover, he cannot do a professional job with integrity unless he believes the assumptions implicit in the technique he selects. Otherwise, the appraisal (or counseling) will not be the result of his own judgment applied to the pertinent facts. Instead it will be the product of a formula which may or may not be plausible in the light of relevant facts." 5

Therefore, it should be clear that the basic role of the computer is as a valuable time-saving device. Its principal role in solving for the internal rate-of-return is to eliminate the hand calculations on a desk-top calculator which would otherwise be necessary. In addition, it will be shown that the speed and low cost of the computer coupled with its flexibility as a computational device will enable the user to simulate ("try-out") many varied complex situations which would not be simulated if we used the valuable time of the appraiser or his staff. In this way the appraiser is able to provide his client
with more accurate and more complete results of his analysis. Such information is the stuff out of which better investment decisions are made. The computer enables you to alter: tax rate assumptions, financing terms, annual projected incomes, selling price, holding periods, depreciation, etc., because it interpolates to the yield resulting from these variations more accurately and faster than other available methods. Since it allows us to vary the input data to find the best results the users output data will enable more rational decision-making, because he will see on the printout the probable results of his different approaches to the investment much more precisely than Ellwood graphic yield analysis. Most important, it would behoove the appraiser or real estate counselor to use the computer because it will provide released time to better serve the client by using his judgment and experience to develop more accurate input information (variables). Investment models are no substitute for judgment in a world of risk and uncertainty. But such computer models do provide more information, much more quickly, and at lower cost. The use of the computer should do much to gain more professional recognition for appraisers and real estate counselors by enabling them to spend more time on: market analysis, site analysis, comparable sales, and other investment data which computers can 'store' but can not interpret.
The Cooper-Phyrr Model and Its Purpose

The purpose of the Cooper-Phyrr model is to estimate the expected after-tax rate of return (expected yield) on apartment building real estate investments. An after-tax internal rate of return on the owner's equity and on total capital invested is calculated by the computer after designating a set of input data and current tax laws.

No originality is claimed for the methodology. In fact, there are more versatile models that have been reported. However, it is felt the simplicity of the model, both as to the input data it calls for, and the output it produces makes it a successful learning tool for novices and those in transition from the older conventional methods of yield calculations. Also, it should be stressed that the model does not appraise the market value of a building. It merely determines expected rates of return upon which an investment decision can be made. The Graaskamp, Shankel, and Farrell models are referred to as sources for those who would pursue valuation rather than investment analysis. Cooper and Zytko have developed similar valuation models which are used at the University of Illinois.

While the computer model is quite simple to use, the accurate estimation of the input values in the model is quite difficult since they depend on forecasts of economic, market, and cost factors over the holding period of the investment. The output
values supplied by the computer are never more accurate than
the input values supplied by the real estate investor. (i.e.,
garbage in = garbage out) Thus, the careful estimation of
input values is of paramount importance, if good investment
decisions are to be made. The process of deriving these esti-
mates will be discussed in detail in our case example which
follows.

Inputs and Outputs in the Model

Table 1 and Figures 1 and 2 on pages 7, 8, and 9 present
the inputs and outputs in the model and flow charts of the
models operation. In the following section, we will set forth
a description of the apartment building which will be the
subject of our analysis, schedule of rents, operating expense
statement, and other pertinent descriptive data necessary to
develop our inputs to the program.

A rate of return (yield) is an abstraction without meaning
if we fail to provide you with accurate and realistic informa-
tion on the environment of the apartment building, its tenants,
operating characteristics, etc. By providing the reader with
this information, we hope to promote a better understanding of
the reliability and accuracy of the computer results. Naturally,
since the input values we estimate are subject to uncertainty
the expected final rate of return values are also subject to
uncertainty, and should be questioned.
Figure 2. Flow Chart: Deterministic Rate of Return Model
Table 1

VARIABLES IN THE DETERMINISTIC RATE OF RETURN MODEL

A. Input Variables
1. BED(1) = number of one bedroom apartments
2. BED(2) = number of two bedroom apartments
3. BED(3) = number of three bedroom apartments
4. FTBED(1) = square feet per one bedroom apartment
5. FTBED(2) = square feet per two bedroom apartment
6. FTBED(3) = square feet per three bedroom apartment
7. RBED(1) = monthly rent per one bedroom apartment
8. RBED(2) = monthly rent per two bedroom apartment
9. RBED(3) = monthly rent per three bedroom apartment
10. GROWR = growth rate in gross rental income over the holding period (compounded)
11. FTCOP = square foot cost of property
12. PERLA = land cost as a dollar amount
13. PEREQ = equity contribution as a percent of total property cost (equity ratio)
14. PEROP = operating cost as a percent of gross total rental income - vacancy expense
15. GROWOC = growth rate in operating costs (compounded)
16. DEPL = depreciable life of building
17. DEPR = depreciation rate (method)
18. TERMA = amortization term of loan
Table 1 (Con't.)

19. RAT = interest rate on loan
20. YTAX = income tax rate
21. CAPTAX = capital gains tax rate
22. SELP = selling price of property (reversion) as a percent of total property cost, average annual appreciation or depreciation
23. NT = holding period of the investment

B. Output Variables
1. FTALL = total square feet of all apartments
2. COSTP = total property cost
3. COSTL = land cost
4. COSTB = building cost
5. DEBT = debt borrowed to finance property
6. ECUITY = original equity invested in property
7. ASDEP = annual depreciation expenses, straight line method
8. ASBOOK = remaining book value of building at end of year, straight line depreciation method
9. ADEP = annual depreciation expense, accelerated method
10. ABOOK = remaining book value of building at end of year, accelerated depreciation method
11. AMORT = annual amortization payment (debt service)
12. AINT = annual interest expense
13. APRIN = annual amortization of principal (reduction in principal amount of loan)
Table 1 (con't.)

14. ARPRIN = remaining principal at end of year
15. ARENT = total annual rental income
16. COSTS = annual operating costs
17. ANIN = annual net income before tax and amortization payment
18. ACASH = annual equity cash flow before tax (after amortization payment)
19. AINC = annual taxable income
20. CASH = after tax equity cash flow
21. REV = selling price of property at end of holding period of the investment (reversion)
22. REVNET = before tax equity reversion (before tax cash flow resulting from sale of property)
23. PBOOK = book value of property at end of holding period of the investment
24. REVTAX = tax on reversion at end of holding period
25. REVFLO = after tax equity reversion (after tax cash flow resulting from sale of property)
26. YIELD = yield on owners equity (internal rate of return)
27. VCASH = after tax cash flow on total capital invested in property (before financing factors are considered)
28. ROR = internal rate of return on total capital invested in property
Investment decisions are never based on true certainty, and it is unwise for anyone to interpret yield calculation as profit realized. Also, because much of the risk in real estate investment is caused by dynamic externalities which are beyond the control of the investor, he is ill-advised to believe that more accurate information on the internal factors which affect his rate of return will, by some alchemy, make his profit more assured.

THE CASTRO-AGNEW APARTMENTS--the subject of our computer analysis

Obviously, the name is fictitious, in fact, by attempting to appeal to such a broad political spectrum the owner may find his apartments appealing to no one, violating a basic tenent of good property management. In order to offend you no further we will henceforth refer to the apartments as the C-A Apartments.

C-A Apartments Data

Property Data

(a) This land parcel is 134 feet in length extending east from the corner of Able and Baker and extending north on Able for a depth of 13? feet. The lot is landscaped with well maintained hybrid grass and a variety of shrubbery and young trees. The lot is rectangular in shape with approximately 50% of the southerly portion occupied by the structure and the northerly 50% covered with asphalt and gravel and used as a parking area by the tenants.
(b) The property is located on the Southeast periphery of the University of Illinois campus, the Digital Computer Lab and Coordinated Sciences Building along with the Television Lab are within one and one-half block. The neighborhood is mixed commercial, residential, and institutional. Residential uses are in transition from old (more than 50 years) large residences to new (less than 15 years) modern apartment buildings. Vacancy rates are extremely low and residential use is at high demand with premium rents possible. A high grade commercial area (Lincoln Square) is about one mile from this location. Supermarkets are within walking distance as is public transportation and public schools. All utility services are adequate and available. All amenities are convenient.

(c) The property is zoned for multi-family use. Present law permits one dwelling unit per 1,000 feet. This structure is a pre-existing non-conforming use to the extent it was built one unit per 750 sq. ft. in all other ways it conforms to the ordinance classification.

(d) The property is devoted to its highest and best use both according to the law and the market which is multi-family residential use built for persons of moderate income and furnished for students due to its proximity to the University.
"I have seen a child... I have spoken to him."

I have seen a child. I have spoken to him. It was strange, but he understood me. He told me a story about a place where magic was real. He said it was a place where dreams come true. I asked him if he would like to go there with me. He smiled and said yes. We talked for a long time about this place and how we could get there. He told me about a special spell that could take us there. I wrote it down and promised to try it. I hugged him goodbye and went home. I'm sure I'll never forget this experience.
(a) **Description and condition of the building** - The building is a 23 unit structure comprised of three floors--eight units on each floor, except for the English Basement where the eighth unit is used for laundry, furnace, hot-water heaters, and other utilities. (For room, hallways, and other area dimensions see the slide provided in class.) Generally, the building is of brick veneer over nailed wood frame (studs 2 X 4 on 16 inch centers) the exterior is of glass curtain walls alternating with a decorative gypsum composition panel. Each apartment has glass exterior paneling and private balcony which is available by sliding door from the living room. The building is contemporary in design with the first floor set 50 percent below grade (English style) to provide maximum daylight to what would otherwise be basement apartments. There are three (3) different size apartments; each provides two bedrooms, bath, kitchen, and living room-dining 'L' shaped room. There are two main entrances to the structure, which lead to separate fire proof stairs at the east and west end of the buildings and on to the common halls which are carpeted throughout. There is access to the stairways from both the front and rear of the building. Each apartment is adequately furnished with modern furniture of high-moderate quality for housing four (4) students; drapes, and carpeting are provided throughout.
Each apartment is equipped as follows: wall-to-wall carpeting with the exception of the kitchen and bath which are covered with vinyl-asbestos tiling; four hollywood beds, four desks, two dressers, one couch, two lounge chairs, a dining table, eight plastic chairs, draperies, floor lamps, coffee table, end table, ceramic floor lamp, four study lamps, refrigerator, range appliances, unit air conditioning, and adequate lighting fixtures.

The walls of the interior are of dry-wall construction, with ceramic tile in the bathroom, and vinyl-coated wood paneling in the living room-dining 'L'. Each bathroom is equipped with modern up-to-date shower-tub combinations and vanity-style lavatories with mirror. The kitchen is small, but well-planned with modern up-to-date appliances and built in wood cabinets. The condition of all interior, exterior, along with furniture fixture, and equipment can be termed excellent with a superior level of maintenance.

The roof is of tar and gravel composition and is in excellent condition. Windows are of steel and aluminum construction with marble sills. Condition excellent. The basement (or first floor below grade) is as follows: footings and foundation are poured concrete. Steel I beams and steel columns support and reinforce the conventional wood frame structure. The building is serviced
by a gas fired, hot water boiler and hydronic baseboard heating system using copper piping throughout. The hot water heater is a gas-fired coil-type instantaneous heater; both are deemed adequate for the building according to engineering standards. All wiring is in compliance with the code consisting of romex in metal tubing.

Each apartment has its own thermostat for heating control, and is individually metered with individual circuit breaker panels. The floor joists are steel frame cross, bridged, on 24 inch centers.

Special feature is the provision of vacuum cleaners for the tenants, garbage disposals in each apartment, and coin laundry facilities in the basement. Each dwelling unit is provided with one reserved parking space in the rear of the building off the street. There is no parking provided for visitors. There are no unlawful conditions, use or occupancy. However, it should be said that some apartments contain more than four students at one time.

The building is only three years old, therefore, depreciation is negligible in view of the high level of maintenance which must be deemed excellent.

There are no auxiliary buildings.

Building has no firewalls.

Storage areas are provided.

Other equipment considered part of the realty: fire
extinguishers, sump pumps, exhaust fans (ea. bathroom and kitchen).

Exterior needs painting, however, part of planned maintenance program to be done this Spring (1972)

Lease Data:

(a) Tenancy is by annual lease, not renewable, adult signer required, must be draft exempt, written contract conventional lease form.

(b) Rent is paid monthly in advance, penalty for late payment by 10th of month 5%; damage deposit of 150% monthly rent required and held in escrow.

Apartments

101, 102, 107, 108, 201, 202, 207, 208 $250.00 per month
103, 206, 301, 302, 307, 308 $253.00 per month
104, 105, 204, 205 $255.00 per month
303, 306 $256.00 per month
304, 305 $256.00 per month
105 (Apt. Manager - amenity rent) $257.00 per month

(c) Fixtures--No separate charge included in gross rent payment.

(d) Utilities--Owner pays: water, heat, garbage collection charges. Tenant pays electricity.

(e) Vacancy--Vacancy factor allowed is only 1% due to unusually tight housing supply conditions prevalent in the area. Under prudent investor standards this is adjusted upward to 4% vacancy reserve.
(f) Leasehold interest is limited to balance of annual lease and does not extend to any improvements of the property.

(g) Lease is terminated only in accordance with the written contract with settlement of escrow damage deposit depending on inventory of condition of the premises.
# INCOME AND OPERATING EXPENSE STATEMENT - 1970*

## INCOME:

<table>
<thead>
<tr>
<th>Description</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gross Possible Rental Income</td>
<td>$69,761.76</td>
</tr>
<tr>
<td>Other Income:</td>
<td></td>
</tr>
<tr>
<td>Laundry</td>
<td>325.77</td>
</tr>
<tr>
<td>Misc. and Interest</td>
<td>132.99</td>
</tr>
<tr>
<td>Insurance Reimbursement</td>
<td>16.00</td>
</tr>
<tr>
<td>Tenant Charges</td>
<td>473.17</td>
</tr>
<tr>
<td><strong>TOTAL INCOME</strong></td>
<td><strong>947.93</strong></td>
</tr>
</tbody>
</table>

## OPERATING EXPENSES:

<table>
<thead>
<tr>
<th>Description</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>Taxes</td>
<td>7,634.73</td>
</tr>
<tr>
<td>Insurance</td>
<td>2,360.00</td>
</tr>
<tr>
<td>Gas &amp; Electricity</td>
<td>2,967.17</td>
</tr>
<tr>
<td>Water</td>
<td>943.37</td>
</tr>
<tr>
<td>Sanitary Hauling</td>
<td>360.00</td>
</tr>
<tr>
<td>Supplies &amp; Materials</td>
<td>1,747.33</td>
</tr>
<tr>
<td>Painting &amp; Maintenance</td>
<td>4,012.73</td>
</tr>
<tr>
<td>Trade/Outside Service</td>
<td>2,155.60</td>
</tr>
<tr>
<td>Management &amp; Administrative</td>
<td>4,246.39</td>
</tr>
<tr>
<td>Accounting &amp; Legal</td>
<td></td>
</tr>
<tr>
<td>Miscellaneous</td>
<td>572.85</td>
</tr>
<tr>
<td><strong>TOTAL OPERATING EXPENSES</strong></td>
<td><strong>27,000.17</strong></td>
</tr>
</tbody>
</table>

## INCOME AFTER OPERATING EXPENSES

**$43,709.54**

## VARIOUS CAPITAL IMPROVEMENT EXPENDITURES:

<table>
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<th>Description</th>
<th>Amount</th>
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</thead>
<tbody>
<tr>
<td>Air Conditioners (2)</td>
<td>467.25</td>
</tr>
<tr>
<td>2nd Floor Curtains</td>
<td>1,139.04</td>
</tr>
<tr>
<td>Furnishings (chairs/cush.)</td>
<td>1,153.08</td>
</tr>
<tr>
<td><strong>TOTAL CAPITAL EXPENDITURES</strong></td>
<td><strong>2,759.37</strong></td>
</tr>
</tbody>
</table>

## NET INCOME AFTER OPERATING EXPENSES AND CAPITAL EXPENDITURES

**$40,950.15**

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1. This includes $3,180 imputed rental income of manager's apartment #105.
2. This figure indicates the fact that much deferred maintenance was done in September of 1970 and tools were purchased since there were none in the building.
3. Sec. #2 - most of this figure shows the painting that was done due to very hard use of apartments.
4. Includes $1,050 paid Carpetland to steam clean carpets.

*This study was commenced in 1970. Be assured that the actual operating expenses for 1972 are within 0.01% of the projections in the computer analysis results for 1972.
Any knowledgeable person should question our operating expenses and our operating expense ratio of .413 on a building four years old. Our explanation is that four (4) active young (usually male) university students are atypical users of property. Therefore, we have provided an unusual repair and replacement program. It provides for a continuous replacement of many items which would normally have a longer life. The shortage of housing in the market enables the owner to adjust the rents upward to provide the income for such a policy of replacement. Further, the location is so close to the campus that it justifies a superior maintenance program to minimize the effect of aging, fair wear and tear in a market with stable and rising values due to the growth of the university. Finally, and most important, these operating expenses are a statement of the owner's actual current practice. He does maintain reserves for replacement in a separate account (drawing interest). He does expect to expend such reserves on the stated items. The low cost of a computer analysis enables us to test the effect on his rate-of-return of such practices and to provide him with the print-out for his review. Because of his wider professional experience it is incumbent on the appraiser to advise the owner of the sensitivity of internal-rate-of-return to changes in the operating expense ratio. Therefore, at the proper place in the input format we will provide for analysis under a more typical use. We derived such operating expense ratios from the Apartment
Building Income-Expense Analysis Annuals as adjusted by local data from apartments in the area.

ALGEBRAIC STATEMENT OF THE COMPUTER MODEL

Since we are indebted to Wendt and Cerf for the basic structure of the Cooper-Pyhrr Model we would like to quote their statement of the algebraic statement in which they acknowledge their debt to Sui N. Wong:

\[
\text{EQUITY} = \sum_{i=1}^{N} \frac{\text{NR}(i) - \text{INT}(i) - \text{PRIN}(i) - \text{TAX}(i) + \text{REV} - \text{CG} - \text{UM}}{(1+r)^i}
\]

NR = Net rent

INT = annual interest payment

PRIN = annual principal payment

TAX = annual tax

REV = Selling price at end of holding period

CG = capital gains tax

UN = remaining amount of unpaid mortgage

r = rate of return

N = holding period

Actually a simplified algebraic expression would have sufficed. But, since our program is an expansion of the above we felt that precision required the more detailed expression. However, it does not matter for the appraiser or real estate counselor does not need to know such equations. Such equations are for programmers to define, manipulate, and refine. The length and efficiency of

*You will note that in our computer analysis we provide for a growth rate in the operating expense ratio which exceeds the growth in expenses resulting from rent increases.
such equations is a matter of mathematical science which for your purposes primarily effects only the "running-time" on the computer. For your purposes as an appraiser, we will assume the Fortran IV program has already been prepared by some consultant or service which makes its income by doing such work. Your concern is with the "Input and Output Data" and the internal logic and consistency of using such input data to determine the output printout. If you are a qualified appraiser your knowledge and experience should enable you to derive, from the information you have gathered on your client's property, all the variables to be "plugged-in" as inputs at the proper place in the computer format. One cautionary comment is worthwhile. Because all computer programmers seek versatility in their programs so that the programs can be used for diverse purposes you will find the programs on occasion require you to fill in 'blanks' which are unnecessary to your calculations but must be completed to maintain logical consistency in the programs. This is a 'fail-safe' device to assure your data has been adapted to the program correctly so that the program can print accurate results. It is believed that careful scrutiny of our adaptation of the C-A Apartments Data to the Cooper-Pyhrr input-output analysis will clarify this point for you by example. You will notice that the model has been designed so that it can be used as a means of testing both existing buildings and calculating the yield of a proposed apartment development with construction cost estimates provided.
The text on the page is not legible due to the quality of the image. It appears to be a page filled with text, possibly a discussion or analysis, but the content cannot be accurately transcribed from the image provided.
HOW TO PREPARE THE INPUT DATA TO PRODUCE THE OUTPUTS

In our program we employed 22 sources of input data. Input 1, 2, 3 are the number of one, two, and three bedroom apartments. Since there are no one or three bedroom apartments in the C-A building we substituted for input 1 and 3 -zero (0). The second input data relates to the number of two bedroom apartments which is 23, so input 2 would be 23.

Inputs 4, 5, 6 have to do with square feet per one, two, and three bedroom apartments. Once again, since there are no one and three bedroom apartments inputs 4 and 6 have similar values of 0. Input 5, which is square feet per 2 bedroom apartments was averaged at 1,000 square feet. The approximation is fair, realistic and typical.

Inputs 7, 8, 9 are the monthly rates of rent per one, two, and three bedroom apartments. Here again inputs 7 and 9 compute the value of 0 because there are no one and three bedroom apartments. If another user were operating this program for a different apartment complex the use of these various outputs may be necessary. Input 8 is the monthly rent per two bedroom apartments for which we used an average of $252.76. This average was taken because some apartments have differing rents, but program logic demanded a single rent statement for all. Of course, the sum of the average rents equals gross income.

Input 10 - Growth Rate in Gross Rental Income Over the Holding Period - (compounded) One of the characteristics of real estate
investment which has a significant effect on the internal-rate-of return is that the charge upon operating income for the debt is an annual constant in the way of a level payment, while gross income may be adjusted upward during the amortization term to reflect not only rising costs but increases in prevailing rents in the area resulting from the rise in the general price level. The owner in the case of the C-A Apartments has made annual increase in the rents for his three years (up to 1972) of ownership ranging from five to seven percent. We attributed his rate of change to some extent a result of the low level of vacancy in the community (2.9% for rentals), and felt that new construction which had been announced would slow down such increases in the near future. Further, our study of various national publications indicated that a more conservative growth factor of 2% per year for such price (rental) increases was more appropriate. Therefore, Input 10 is .025.

Input 11 is the square foot cost of the property which is calculated to be $13.39 per sq. ft. Obviously, if this program were being used as an economic feasibility study based on the contractor's estimates we would use his average square foot cost as a basis of calculation. We can also take the square foot costs from the various building cost calculators as adjusted to the vicinity if construction costs or purchase price are unknown. Care must be taken to provide for an all-inclusive property cost amount covering labor, materials, interim financing, fees, land,
and other development costs. If purchase price is known it is a simple matter to convert it to a square foot cost.

Input 12 is a statement of the land costs as a dollar amount, which in this case is $42,000. Such figure may be derived from: the books of account; the actual land purchase price; where not known, it may be necessary to derive it from your data bank or the expert opinion of qualified appraisers in the area; or in a feasibility study you may wish to use a fair average cost of land per dwelling unit as known in your area. In any case it must be stated for it is part of the all inclusive property cost per square foot set forth in Input 11.

Input 13 - is Equity Contribution as a percent of total property cost. In our case we have used .25 with 75% mortgage financing and .10 when 90% financing might be employed at the request of the owner. He did not wish to reveal his actual equity contribution to the purchase price and wanted to see the sensitivity of the internal-rate-of-return to the leverage of the lower equity contribution at 90/10. One can readily see how variations on this critical element could produce much valuable information for a would-be syndicator.

Input 14 - Operating costs stated as a percent of total rental income. We have already commented on this input variable in this article. We will input 41.3% at the owner's request representing 1970 his first year.
Input 15 - Growth Rate in Operating Costs Ratio (compounded)-

As can be seen from Input 14 we have made increases in operating expenses a function of increases in rental income resulting from the rise in general price levels .025. Obviously, this may not be the case. It is quite possible that operating expenses may increase at a rate entirely independent of the owner's increases in rents. In fact, some areas have reported recent increases in operating expenses of 8 - 15 per cent of the previous years expenses. Input 15 provides the appraiser with the opportunity to treat operating expenses as a variable with its own rate of change. Therefore Input 15, in this case, is .005.

Input 16 - states the depreciable life of the building. Usually the owner has already adopted a useful life as in this case - 40 years. Since the building was three years old at time of purchase he could certainly have used 37 years, and the author is of the opinion that this type of construction would have justified the use of an original 33 year useful life or remaining life at time of purchase of 30 years. It is a simple matter to 'plug-in' the proper depreciable life. Of course, some FHA financing suggest a useful life the same as the amortization term or longer (i.e. 40 - 50 years). However, it is believed that limiting the purchasers of existing buildings to a maximum depreciation rate of 125% of remaining balance will place considerable downward pressure on useful life.
Input 17 - Chosen Depreciation Rate Method. We used 100% straight-line; 150% declining balance; and 200% declining balance. 125% is an implicit interpolation between straight line and 150%. We did use Sum-of-the-Digits only as a matter of convenience. It is a simple matter to make it a part of our study for the owner.

Input 18 - states the Amortization term of the debt. In this case we used 20 years. For a Sec. 236 project you would use 40 years. Though the internal-rate-of-return is highly sensitive to this input because a shorter amortization term (e.g. 10 years) will substantially affect annual net cash flows and tax liabilities we decided to limit the analysis on this item to a fairly typical 30 years for an existing relatively new project.

Input 19 - Interest rate on the loan. We used in our calculations 6, 6.5, 6.75, 7, 7.5, 8, 8.5, 9.0. Obviously, we have used many variations of this crucial variable. Because of the instability of mortgage credit and the recent wide variations in mortgage contract interest rates we felt this variable was of greatest interest to the investor. It permits a valuable test of the interaction of leverage (i.e. 75 - 25 and 90 - 10 mortgage) with the interest cost of the debt and its effect on the rate-of-return.

Note: Computer programs, of course, have the flexibility to provide additional inputs for junior-lien mortgages and chattel
mortgages on furniture, equipment, carpets, etc. All that would be necessary is to add an additional input to fix an additional charge on the cash flow at a different amortization term with another input for the different interest rates which relates there to. The Cooper-Pyhrr model actually has ten (10) depreciation elements and three (3) amortization rate and term elements thus permitting all practical variations to be used.

Input 20 - The federal income tax rate - We chose .35 because it was felt that the equity contribution required by this project would tend to require persons in the higher tax brackets. Further, it provided an approximation of the corporate income tax rate to simulate corporate ownership. Clearly, this rate can and should be varied in accordance with the effective income tax rate of the investors. Some scholars suggest you should have the highest bracket for the taxpayer in question because it is that bracket which is most probably tax sheltered by the depreciation. Because our experience has indicated that the impact of the 'losses' resulting from early year depreciation and interest expense cuts deeper than the highest (i.e. marginal) bracket the more proper thing to do would be to use the taxpayer's effective tax rate which is an approximation of the ratio of his total tax to his estimated adjusted gross income less deductions for personal exemptions and expenses. A source for such a rate could be his estimated tax return for the current year as adjusted by recent expectations. It should be clear by now that the programmer could
provide another Input for the State Income Tax rate if that is a significant factor in the appraiser's market. Certainly if it is an additional burden on the cash flow for it does affect the internal-rate-of-return and it would be desirable to provide for such a charge.

Input 21 - Capital Gains Tax Rate - We used 30 percent. This is the maximum capital gain tax on the net reversion for corporations under the Tax Reform Act and apply to sales now. In actuality non-corporate tax payers are paying 32.5% in 1971 and will pay 35% in 1972. By 1975 the rate will be the same as the individuals personal income tax rate for one-half of the gain, while the other half on the gain will be subject to a 10% additional surcharge as a tax preference item.

Of course only the net gain is subject to the tax. The net gain is the difference between the ultimate selling price and cost basis as adjusted by the depreciation reserve which has reduced the cost basis each year over the holding period. It is noted here that sales made prior to 16 years and 8 month from the date of construction or purchase are subject to an "excess depreciation" surcharge which causes all the depreciation which was in excess of a straight-line rate to be charged back to ordinary income in the year of the sale and taxed at the taxpayers ordinary income rate. We have provided for such higher tax rates for our shorter holding periods of one to fifteen years in our program.
Input 22 - Expected Selling Price of the Property (Reversion) as an annual average app./dep. of original total property cost. The problem here is forecasting the rate of change in real estate values likely to occur over the holding period. Obviously, such an exercise of judgment can only be subjective. However, it is felt that a qualified appraiser's knowledge better equips him to make such judgments than the average intelligent layman who does not have the appraiser's special knowledge of the trends of market sales in the vicinity. It was our decision to use a rate of gain in this university-dominated city of .035% per annum. Though somewhat controversial, we chose to make our app./dep. of the reversion to be incremental rather than compounded because we believe that is the way appraisers view the real world. (Example: a 20% appreciation in 10 years is a flat 2% per year rather than compounded.) Certainly, a property might depreciate in value over time due to the effect of the neighborhood, aging, and other reasons. But, whatever the ultimate judgment is; 'plugging it into the computer' is the easy part of the problem.

Input 23 - Holding Period of the Investment. The variables can be any desired. We used in our program each year for 20 years to seek the optimal holding period in relation to the yield (internal-rate-of-return). It is in this regard that computer analysis is superior to an experienced appraiser using his professional judgment and the crude tools of tables and a hand calculator.
It is a waste of human skills to do the laborious task of the repetitive calculations necessary to determine the yields, by interpolation, for all these holding periods or to approximate them by using a french curve tool for graphic analysis. The large number of variables and the complex impact of: excess depreciation, preference taxes, capital gains taxes, etc. beg for the low cost 'idiot' computer which will do the drudgery of calculating the 'true' yield which takes into account all these many factors. The computer is ideally suited for the task. A review of our tables of results will show that actual results do not necessarily fall where our intuition would guide us in the absence of such information.

ANALYSIS OF C-A OUTPUT DATA - assuming constant income and expenses and no change in value.

Table 2.

Table 2 presents internal rates of return on assets and equity, assuming constant rental income, constant operating costs, and a selling price of the property that is equal to the original property cost. Other input data is listed below the Table. Twenty holding periods are analyzed, with interest rates ranging from 6% to 9% and depreciation rates of 100%, 150%, and 200% and sum of the digits. The percentage of debt used to finance the property acquisition is 75% and 90% respectively, both assuming a 20 year amortization period.
Table 2(a)
C-A Apartments, Champaign, Illinois

DCF Rate of Return on Equity and Total Capital Invested - 75% mortgage - 25% Equity - No Growth Case
Symbols for Depreciation Method Used: s.l. = straight-line; 1.50 = 150% declining balance; 2.00 = 200% declining balance and
3.0 = sum of the digits method  Mortgage Contract interest rate is as indicated on the next line below -

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Note: This table provides for '0' growth rates in: rental income over time = '0'
operating expenses over time = '0'
the value of the reversion = '0'
ammortization term is 20 years

Source: Cooper-Pyburn analysis

Symbol legend: DCF = Discounted Net Cash Flow after Taxes
EY = True Yield on Equity by Internal Rate of Return
TA = Internal Rate of Return on Total Capital Assets
### Table 2 (b)

**DCF Rate of Return on Equity and Total Capital Invested**  
- **90% mortgage**  
- **10% Equity**  
- **No Growth Case**  

**Symbols for Depreciation Method Used:**  
- s.1. = straight-line  
- 1.50 = 150% declining balance  
- 2.00 = 200% declining balance  
- 3.0 = sum of the digits method

**Mortgage Contract interest rate is as indicated on the next line below**

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**Note:** This table provides for '0' growth rates in:  
- rental income over time = '0'  
- operating expenses over time = '0'  
- the value of the version = '0'  
- amortization term is 20 years

**Source:** Cooper-Pyburn analysis

**Legend:**  
- **DCF** = Discounted Net Cash Flow after Taxes  
- **EY** = True Yield on Equity by Internal Rate of Return  
- **TA** = Internal Rate of Return on Total Capital Assets
A number of interesting observations can be made from the output data in this Table:

(1) The after-tax rates of return on equity for conventional 75% mortgage financing range from 12.18% (9% interest rate, 100% depreciation, 20 year holding period) to 23.01% (6% interest rate, sum of digits depreciation, 3 year holding period). In contrast, after-tax rates of return on assets (ROR) are substantially lower in all cases, and is not influenced by the amount of financing or the cost of financing. Thus, given any interest rate, and percent of mortgage financing, the ROR varies between 7.73 and 9.03 and depends on the holding period of the investment and the depreciation method used, but never on the degree of leverage employed or the amortization period. The relatively low over-all rates of return (on a property considered to be very successful) is indeed of great interest, perhaps one reason why real estate analysts stress equity YIELD as the primary rate of return indicator in real estate instead of the ROR measure! The ROR does not seem to tell us much for decision making purposes but is extremely relevant to mortgage lender fearful of default.

(2) Given the higher degree of leverage financing in the 90-10 case, equity yields rise substantially. In contrast, and as expected, the rate of return on assets maintains the same structure as in the 75% financing case. Also, the equity yield rises more substantially for cases of low interest rates than
high interest rates, thus illustrating the loss of leverage benefits as interest rates rise. For example, at a 6% interest rate, 3 year holding period, the rate of return on equity approximately doubles when the percent of mortgage financing is increased from 75% to 90%. At 9% interest, the increase is less. However, the dispersion of the expected rates of return on equity are also greater between successive holding periods as the debt ratio increases, thus significantly changing the structure of rates of return on equity over different time horizons.

(3) Higher equity yields are obtained as more accelerated rates of depreciation are employed, with the absolute differences becoming larger as more leverage financing is applied. For example, at 6% interest rate, a ten year holding period, and 75% mortgage financing, the equity yield rises from 17.02% to 18.48% when double declining depreciation (200%) is employed rather than straight line depreciation. When 90% mortgage financing is employed, given the same interest rate, holding period, and depreciation schedules, the equity yield rises from 30.0 to 35.0. Had a 40 year amortization period been used, the rate of return might have risen from 30.0 to approximately 40.0.

(4) If we define the optimum holding period as that holding period over which the rate of return is the highest, then some interesting observations can be made about the optimum holding period of the investment, especially when one compares ROR to
YIELD. If one wishes to maximize his YIELD (rate of return on equity), then short holding periods tend to be optimum, with exceptions occurring at higher interest rates and 75% mortgage financing. In contrast, if one wishes to maximize ROR then in all cases a long holding period is desirable. (Both optimum ROR's and YIELDS are underlined in the tables. The computer calculated to the fifth decimal. We rounded for publication.) The conflict in results resulting from the use of one rate of return measure versus the other is readily apparent. If one uses the YIELD approach, and makes investment decisions which have as their objective the maximization of this return, then frequent property turnover will tend to occur. (Transaction costs, business and financial risks, and opportunity costs are being ignored for the moment, of course.) On the other hand, if one employs the ROR as a decision making criterion, as do many capital budgeters, then long holding periods will tend to be the rule if the decision maker has confidence in the validity of the data.

Unfortunately, the 90% case in Table 2 would probably not be feasible at high interest rates (9%). Despite rates of return between 16.75 - 33.34%, the yearly amortization payment is so large that yearly 'negative' equity cash flows occur frequently. In such cases the reversionary cash flow constitutes the essence of the rate of return in these cases, and the investor is faced with out-of-pocket costs each year until he sells his project and realizes his capital gain. Even if the
investor would be willing to go along with a situation such as this, the lenders would undoubtedly express little enthusiasm over the project. In contrast, when longer amortization periods are allowed and/or inflation factors are added, this project becomes more feasible at high interest rates. The following analysis attempts to inject inflation factors into our analysis.

Table 3(a) and 3(b)

Three price or inflation factors have been built into the rate of return model, as previously mentioned, and could be manipulated by the analyst as he sees fit. (Note that increased housing demand is assumed here to be an inflation factor.) They are again:

(a) Yearly increases in rental income (compounded)--as mentioned previously this was established at 2.5%.

(b) Yearly increase in total property value--this was established at 3½%. (See Roy Wenzlick, Real Estate Analyst, plus adjustments for local factors: slowdown of University growth, increased housing supply, etc.)

(c) Yearly increase in the operating cost percentage (compounded)--given apartment experience data collected by the Institute of Real Estate Management Experience Exchange Committee for similar type structures, it was estimated that the yearly increase in this percentage was approximately .005. While this data was collected on a cross-sectional basis, it is estimated
### Table 3 (a)

C-A Apartments, Champaign, Illinois (23 units)

DCF Rate of Return on Equity and Total Capital Invested - 7% mortgage - 25% Equity - with growth rates 6.5% per annum

Mortgage Contract interest - 6.0% per annum

Depreciation Method indicated below:

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Note: This table provides for certain growth rates:
- Rental increase = 2.5% per annum (compound)
- Operating Expense Ratio = .5% (compound)
- Reversion = 3.5% (incremental)
- Amortization term is 20 years

Source: Cooper-Pyatt Analysis

Symbol legend:
- DCF = Discounted Net Cash Flow after Taxes
- EY = True Yield on Equity by Internal Rate of Return
- TA = Internal Rate of Return on Total Capital Assets
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Note: This table provides for certain growth rates: Rental income - 2.5% per annum (compound), Operating Expense Ratio - .003 (compound), Reversion - 3.5% (incremental), amortization term is 20 years.

Source: Cooper-Pyburn Analysis

Symbol legend: DCF = Discounted Net Cash Flow after Taxes
EY = True Yield on Equity by Internal Rate of Return
TA = Internal Rate of Return on Total Capital Assets
that this structure of increases in operating costs will be consistent with the actual operation of our building in the future.

As the inflation factor increases yearly rents, operating expenses rise at an increasing rate due to (a) an increasing operating cost percentage applied to (b) an increasing rental income each year. The effect is to increase yearly operating expenses at a rapid rate. This expectation is consistent with recent operating cost trends in the area and nationally. Also consistent with recent real estate trends is the assumed rise in property value in the future.

Numerous authors assume away the effect of these inflation factors. For instance, Soelberg and Stefaniak discount the property value and income-expense increases (decreases) in the following manner:

. . . Taking into account (1) an expected inflationary price rise, (2) an average estimate of neighborhood deterioration, (3) increased market-demand pressures due to population growth, and (4) increasing economic and functional obsolescence of the property, it is reasonable to assume that the market value's future growth rate is negligible. . .

Conflicting pressures from inflationary price rises, cost increases, and functional
obsolescence of the improvements could also yield an assumed average rate of growth of about zero percent yearly--net operating income increases.

(Appraisal Journal, April 1970)

The authors, in our opinion, omit one very important consideration; that is, that an investor who mortgages a large proportion of his investment has equal yearly (or monthly) amortization payments to make. As rents rise due to increased demand and overall price level rises, the net income to the investor also rises, despite a rise in the percentage of operating cost. Thus, while operating costs rise at an increasing rate, the amortization payment is constant, resulting in a rising amount of net income to the investor. This has been a typical situation in the Champaign-Urbana area in the 1960's and is expected to continue in the 1970's, assuming the building is within 0-5 years of age. A similar situation is expected for property value appreciation. 9

The inflation factors, when incorporated into the rate of return analysis, have some interesting implications relating to the rate of return and optimum holding periods.

Table 3(a) and 3(b) presents the output results for the rate of return on assets and equity for the two different types of financing, interest rates, depreciation rates, and holding
periods. The differences in these rates of return, which include the three inflation factors mentioned are significant. Not only are the rates of return high compared to those in Table 2, but the optimum holding period is three years in every case, employing the rate of return on equity criterion, and 20 years in every case employing the rate of return on assets criterion.

The after-tax rates of return on equity for conventional 75% mortgage financing range from 16.13% (9% interest rate, 100% depreciation, 20 year holding period) to 28.78% (6% interest rate, sum of digits depreciation, 2 year holding period). The rates of return on assets have increased from a range of 9.48% to 10.75%, again with an extremely long optimal holding periods due to the rising ROR over longer time periods. At 90% mortgage financing, the rates of return on equity range from 23.02% to 61.75%, indicating the significant effects of leverage combined with inflation on the expected rate of return on equity.

It would certainly appear, given this data, that real estate investors have a built in multiplier in their rate of return during a period of significant prices rises in the economy, or in a superior location in a locality characterized by high growth in housing demand. In addition, the greater the degree of inflation or leverage (debt) financing employed, the greater the multiplier effect in general.

However, as previously stressed, expected rates of return are not the same thing as realized rates of return. A slowdown
in the growth of the University coupled with persistent overbuilding in the community (which is now tending to occur may certainly lower the actual rate of return below the expected. In contrast continued inflation in conjunction with efficient management, the superior location of the building on campus, and superior maintenance and upkeep of the premises, may raise the realized rate of return on equity and assets above the expected.

It is worthy to note, then, that we are dealing with expected-mean-value after-tax rates of return, and not ranges of expected values which could be outputted in a probabilistic rate of return model. While the later type of model is most desirable, so that risk analysis can be incorporated into our analysis, the rate of return model is a necessary prerequisite. Furthermore, accurate forecasting techniques, coupled with a good data base, are prerequisites of a useful rate of return analysis; neither of these appear to be prevalent in the housing investment field, and perhaps should receive the greatest degree of attention from real estate decision makers.

This case study has demonstrated the importance of the appraiser's expertize in selecting input data; the great value of computer analysis in simulating differing investment approaches to the subject property to produce precise yield analysis; and demonstrated that yield analysis by a deterministic
rate-of-return model is net risk analysis for true forecasting techniques though it is the best available method to evaluate a project.
FOOTNOTES


7. op. cit. Footnote 6.


9. Professors Cooper and Pyhrr have an unresolved disagreement. Cooper accepts Pyhrr's position that corporate financial analysts do not deflate earnings because of inflationary effects eroding the 'real dollar' value of the capital investment. In effect, nominal dollar increases in earnings per share are treated as though they are true gains. Professor Cooper suggests that increases in profits resulting from rent and reversionary increases due to inflation are somewhat illusory, except to the extent such money is used to amortize the debt which is in constant dollars.